QUANTIZATION OF $N=1,2$ SUPERPARTICLE WITH IRREDUCIBLE CONSTRAINTS

E.R. NISSIMOV [1] and S.J. PACHEVA [1]

CERN, CH-1211 Geneva 23, Switzerland

An action for the $D=10$ superparticle with $N=1,2$ supersymmetry, containing irreducible (functionally independent) Lorentz covariant constraints only, is proposed. The key ingredient is the enlargening of the usual superspace to include additional bosonic harmonic coordinates which transform as Lorentz spinors and are pure gauge degrees of freedom. Upon quantization the extended superparticle provides a Lorentz covariant on-shell description of the linearized $D=10$ type IIb supergravity. The first-quantized BRST charge contains a multighost interaction term which is inevitable when the functional dependence of the reparametrization and the local fermionic symmetry constraints is correctly taken into account.

1. Recently there has been much interest in analyzing Siegel's superparticle [1–3] especially in view of the fact that, representing a "zero-mode" approximation to the covariant Green–Schwarz superstring [4], it shares some of the fundamental features of the latter, in particular, the local fermionic symmetry.

The Siegel superparticle action [1]

$$S= \int d\tau(p_\mu \partial_\tau x^\mu - p_\theta^\alpha \partial_\tau \theta_\alpha - \lambda p^2 - \psi^\alpha \tilde{p}_{\alpha\beta} d^\beta),$$

where λ and ψ^α are Lagrange multipliers and the Grassmann spinor θ_α is Majorana or Majorana–Weyl (MW) (in $D=10$; we shall concentrate on this case here), possesses two first-class constraints [11]:

$$p^2 =0 \quad \text{and} \quad \tilde{p}_{\alpha\beta} d^\beta \equiv \tilde{p}_{\alpha\beta}(ip_\theta^\beta - \tilde{p}^{\beta\gamma}\theta_\gamma)=0.$$

However, a systematically overlooked property of these constraints is that they are functionally dependent, i.e. they form a reducible set in the sense of ref. [5] [12].

Then one is left with the following two alternatives:

(i) either, one must separate the half independent constraints from $\tilde{p}_{\alpha\beta} d^\beta$ which cannot be performed in a Lorentz-covariant manner, since $\tilde{p}_{\alpha\beta} d^\beta$ is a $D=10$ MW spinor (there are no Lorentz-covariant objects with half the number of components of a MW spinor);

(ii) or, one has to employ the general scheme of Batalin–Fradkin–Vilkovisky (BFV) [5] for systems with reducible constraints. In this case, the price for working with reducible constraints is the introduction, for each functional dependence among the latter, of additional ghosts which enter the BRST [6] charge and the BFV hamiltonian [7]. However, one can easily deduce, that in this case as well, due to the MW nature of the constraint $\tilde{p}_{\alpha\beta} d^\beta$, it is impossible to retain manifest Lorentz invariance maintaining simultaneously a finite degree of reducibility L of the system.

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[1] Permanent address after January 1, 1987: Institute of Nuclear Research and Nuclear Energy, Boulevard Lenin 72, 1784 Sofia, Bulgaria.

[11] The $D=10$ and $D=8$ spinor conventions are explained in the Appendix.

[12] The set of first-class constraints $\{\Phi_{\alpha_0}\}$ possesses a degree of reducibility L if the following relations hold:

$$\Phi_{\alpha_0 (1)} Z^{\alpha_0}_{\alpha_1}=0, \quad {}_{(1)}Z^{\alpha_0}_{\alpha_1 (2)}Z^{\alpha_1}_{\alpha_2}|_{\Phi=0}=0, \quad ..., \quad {}_{(L-1)}Z^{\alpha_{L-2}}_{\alpha_{L-1} (L)}Z^{\alpha_{L-1}}_{\alpha_L}|_{\Phi=0}=0, \quad \alpha_k=1, ..., m_k,$$

where ${}_{(1)}Z^{\alpha_0}_{\alpha_1}, ..., {}_{(L)}Z^{\alpha_{L-1}}_{\alpha_L}$ are functions of the canonical variables which do not identically vanish on the constraint surface $\Phi_{\alpha_0}=0$.

Thus, consistent Lorentz-covariant canonical quantization of Siegel's superparticle action, properly accounting for the reducibility of the constraints, is impossible without some further modification.

In this letter we propose a formalism which overcomes the above mentioned difficulties. The key ingredient is the introduction of bosonic Lorentz-spinor harmonics v_α^A, $\tilde{v}_A^\alpha$ subject to the constraint $v_\alpha^A \tilde{v}_B^\alpha = \delta^A{}_B$ as additional dynamical variables. The enlarged $N=1, 2$ superspace coordinates are now labeled by $(x^\mu, \theta_\alpha^{1,2}, v_\alpha^A, \tilde{v}_A^\alpha)$. (In what follows we shall directly consider the more general case of the point particle with $N=2$ supersymmetry). The spinor harmonics serve as "bridges" converting the SO(1, 9) MW spinor indices α into a pair of SO(8) spinor indices $A=(a, \dot{a})$ corresponding to the (s), (c) representations, respectively. The inclusion of v_α^A, $\tilde{v}_A^\alpha$ is crucial in order to separate in a Lorentz-covariant way the 8 independent components of the MW spinor constraint $\dot{p}_{\alpha\beta} d^\beta$. We also include additional first-class constraints into the superparticle action to guarantee that v_α^A, $\tilde{v}_A^\alpha$ are pure gauge degrees of freedom.

The present formulation is very much inspired by the harmonic superspace approach to extended supersymmetric (gauge) theories in $D=4$ [8] and by the recently developed covariant light-cone superspace in $D=10$ [3]. However, the latter differs substantially from the present one in two respects: (a) the additional harmonics introduced in ref. [3] are Lorentz vectors instead of Lorentz spinors; (b) the physical content of the theory in ref. [3] is different. Moreover, the constraints of the modified superparticle action proposed there once again form a reducible set.

In section 2 an action with irreducible constraints for the extended superparticle is proposed. In section 3 the covariant first quantization a la Dirac is accomplished. After imposing on-shell of a suitable covariant reality condition on the superfield wave function it is demonstrated that the theory coincides with the linearized $D=10$ IIb supergravity, known in the light-cone formalism [9] (cf. also Townsend [2]). In section 4 the first-quantized BRST charge is computed and found to contain a multighost term, i.e. the $N=1, 2$ superparticle is a constrained system of rank two in the sense of ref. [7].

2. Let us consider the following classical action for the $D=10$ extended superparticle described by coordinates $(x^\mu, \theta_\alpha^1, \theta_\alpha^2, v_\alpha^A, \tilde{v}_A^\alpha)$:

$$S_{N=2} = \int d\tau \left(p_\mu \partial_\tau x^\mu - \sum_{k=1,2} p_\theta^{k\alpha} \partial_\tau \theta_\alpha^k + p_{v,A}^\alpha \partial_\tau v_\alpha^A + \partial_\tau \tilde{v}_A^\alpha p_{\tilde{v}}{}_\alpha^A - H \right), \tag{1}$$

$$H = \lambda p^2 + \rho^a \mathscr{D}_a + \mathscr{D}^\alpha \psi_\alpha + \Lambda_{AB} \mathscr{D}^{AB}, \qquad \Phi_\alpha{}^\beta \equiv v_\alpha^A \tilde{v}_A^\beta - \delta_\alpha{}^\beta = 0, \quad \chi_\alpha{}^\beta \equiv v_\alpha^A p_{v,A}^\beta + p_{\tilde{v}}{}_\alpha^A \tilde{v}_A^\beta = 0. \tag{2,3}$$

Here $\theta_\alpha^{1,2}$ are $D=10$ MW (left-handed) Grassmann spinors. The bosonic Lorentz-spinor harmonics $v_\alpha{}^{(a,\dot{a})}$, $v_{\alpha(a,\dot{a})} = (C_8^{-1})_{\alpha\beta} \tilde{v}_{(a,\dot{a})}^\beta$ transform as $D=10$ MW left- and right-handed spinors with respect to the indices α, $\dot{\alpha}$ and transform as SO(8) (s)-, (c)-spinors with respect to the indices $(a, \dot{a}) = A$. The hamiltonian is taken as a linear combination of irreducible first-class constraints with the following notations:

$$\mathscr{D}_a \equiv \tilde{v}_a^\alpha \dot{p}_{\alpha\beta}(ip_\theta^{1\beta} - \tilde{p}^{\beta\gamma} \theta_\gamma^2), \quad \mathscr{D}^\alpha \equiv ip_\theta^{2\alpha} - \tilde{p}^{\alpha\beta} \theta_\beta^1, \quad \mathscr{D}^{AB} \equiv p_v^{\alpha A} v_\alpha^B - \tilde{v}^{\alpha A} p_{\tilde{v}}{}_\alpha^B. \tag{4}$$

Let us emphasize that with the help of v_α^A, $\tilde{v}_A^\alpha$ we have covariantly separated the Siegel fermionic constraint into two parts:

$$\dot{p}_{\alpha\beta}(ip_\theta^{1\beta} - \tilde{p}^{\beta\gamma} \theta_\gamma^2) = v_\alpha^a \mathscr{D}_a + v_\alpha^{\dot{a}} \mathscr{D}_{\dot{a}},$$

and the second part $\mathscr{D}_{\dot{a}}$:

$$\mathscr{D}_{\dot{a}} \equiv \tilde{v}_{\dot{a}}^\alpha \dot{p}_{\alpha\beta}(ip_\theta^{1\beta} - \tilde{p}^{\beta\gamma} \theta_\gamma^2) = -[(\tilde{S}^{-1})_{\dot{a}b} v_\alpha^b(ip_\theta^{1\alpha} - \tilde{p}^{\alpha\beta} \theta_\beta^2)]p^2 - (\tilde{S}^{-1} \tilde{R})_{\dot{a}}{}^b \mathscr{D}_b$$

$$(\tilde{R}^{\dot{a}b} \equiv v_\alpha^{\dot{a}} \tilde{p}^{\alpha\beta} v_\beta^b, \ \tilde{S}^{ab} \equiv v_\alpha^a \tilde{p}^{\alpha\beta} v_\beta^b), \tag{5}$$

whose functional dependence on the remaining constraints p^2 and $\mathscr{D}_a$ is explicitly displayed in (5), is excluded from (1), (2).

The action (1) is manifestly invariant under $N=2$ supersymmetry transformations

$$\delta_{ss}x^\mu = -\mathrm{i}[\epsilon^1_\alpha(\tilde{\sigma}^\mu)^{\alpha\beta}\theta^2_\beta + \epsilon^2_\alpha(\tilde{\sigma}^\mu)^{\alpha\beta}\theta^1_\beta], \quad \delta_{ss}\theta^{1,2}_\alpha = \epsilon^{1,2}_\alpha, \quad \delta_{ss}\tilde{v}^\alpha_A = \delta_{ss}v^A_\alpha = 0,$$

where $\epsilon^{1,2}_\alpha$ are $D=10$ MW spinors.

For $N=1$ supersymmetry the action (1) and the hamiltonian reduce to

$$S_{N=1} = \int \mathrm{d}\tau(p_\mu\partial_\tau x^\mu - p^\alpha_\theta\partial_\tau\theta_\alpha + p^\alpha_{v,A}\partial_\tau v^A_\alpha + \partial_\tau\tilde{v}^\alpha_A p^A_{\tilde{v}\,\alpha} - H), \quad H = \lambda p^2 + \rho^a\tilde{v}^\alpha_a \dot{p}_{\alpha\beta}d^\beta + \Lambda_{AB}\mathcal{D}^{AB},$$

eq. (3) remaining the same.

The constraints $\Phi_\alpha{}^\beta, \chi_\alpha{}^\beta$ (3) are second class and form a conjugate pair which commutes with all first-class constraints in eq. (2) [13]:

$$\{\Phi_\alpha{}^\beta, \chi_\gamma{}^\delta\}|_{\Phi,\chi=0} = 2\delta_\alpha{}^\delta\delta_\gamma{}^\beta, \quad \{\mathcal{D}^{AB}, \Phi_\alpha{}^\beta\} = \{\mathcal{D}^{AB}, \chi_\alpha{}^\beta\} = 0, \quad \text{etc.}$$

Thus, any Poisson bracket relations involving the first-class constraints, in particular, their algebra remain unchanged after introducing the corresponding Dirac brackets:

$$\{\mathcal{D}_a, \mathcal{D}^\alpha\} = -2\mathrm{i}\tilde{v}^\alpha_a p^2, \quad \{\mathcal{D}_a{}^b, \mathcal{D}_c\} = \delta_c{}^b\mathcal{D}_a, \quad \{\mathcal{D}_A{}^B, \mathcal{D}_C{}^D\} = \delta_C{}^B\mathcal{D}_A{}^D - \delta_A{}^D\mathcal{D}_C{}^B,$$

$$\{\mathcal{D}_{\dot{a}}{}^b, \mathcal{D}_c\} = -\delta_c{}^b[(\tilde{S}^{-1})_{\dot{a}d}v^d_\alpha(\mathrm{i}p^{1\alpha}_\theta - \tilde{p}^{\alpha\beta}\theta^2_\beta)p^2 + (\tilde{S}^{-1}\tilde{R})_{\dot{a}}{}^d\mathcal{D}_d], \quad \text{rest}=0. \tag{6}$$

Now, let us briefly comment on the significance of the Lorentz-spinor harmonics $v^A_\alpha, \tilde{v}^\alpha_A$. From the constraint $\Phi_\alpha{}^\beta = 0$ (3) one immediately sees that the objects

$$\Pi_{L\alpha}{}^\beta \equiv v^a_\alpha\tilde{v}^\beta_a, \quad \Pi_{R\alpha}{}^\beta \equiv v^{\dot{a}}_\alpha\tilde{v}^\beta_{\dot{a}}, \quad \Pi_{L\alpha}{}^\beta + \Pi_{R\alpha}{}^\beta = \delta_\alpha{}^\beta, \tag{7}$$

are Lorentz-covariant projectors for the $D=10$ MW spinors. Thus (7) are precisely the covariant analogues of the usual light-cone projectors:

$$-\tfrac{1}{2}(\sigma^-\tilde{\sigma}^+)_\alpha{}^\beta = \Pi^{(8)\beta}_{L\alpha}, \quad -\tfrac{1}{2}(\sigma^+\tilde{\sigma}^-)_\alpha{}^\beta = \Pi^{(8)\beta}_{R\alpha}, \tag{7'}$$

where $\Pi^{(8)}_{L,R}$ are the $D=8$ chiral projectors.

Under group transformations generated by the GL(16; $\mathbb{R}$) algebra of $\mathcal{D}^{AB}$ (6) v^A_α and $\tilde{v}^\alpha_A$ transform as follows:

$$\exp\{\omega\mathcal{L}_\mathcal{D}\}v^A_\alpha = G^{-1}(\omega)^A{}_B v^B_\alpha, \quad \exp\{\omega\mathcal{L}_\mathcal{D}\}\tilde{v}^\alpha_A = \tilde{v}^\alpha_B G(\omega)^B{}_A, \tag{8}$$

where

$$G(\omega)^A{}_B = (\exp\omega)^A{}_B, \quad (\omega\mathcal{L}_\mathcal{D})F(v,\tilde{v}) \equiv \omega^A{}_B\{\mathcal{D}_A{}^B, F(v,\tilde{v})\}.$$

Thus, using (8) one can always impose the following Lorentz-noncovariant gauge on $v^A_\alpha, \tilde{v}^\alpha_A$:

$$v^a_\alpha = -\tfrac{1}{2}(\sigma^-)^a_\alpha, \quad v^{\dot{a}}_\alpha = -\tfrac{1}{2}(\sigma^+)^{\dot{a}}_\alpha, \quad \tilde{v}^\alpha_a = (\tilde{\sigma}^+)^\alpha_a, \quad \tilde{v}^\alpha_{\dot{a}} = (\tilde{\sigma}^-)^\alpha_{\dot{a}}. \tag{9}$$

In particular, in the gauge (9) the covariant projectors (7) reduce to the usual light-cone ones (7'). This non-covariant gauge will be useful for establishing the on-shell equivalence of the covariantly quantized extended superparticle with the linearized $D=10$ IIb supergravity in the light-cone formulation [9].

3. Let us now consider the covariant first quantization of (1) a la Dirac. The superfield wave function $\Phi = \Phi(x, \theta^1, \theta^2, v, \tilde{v})$ must satisfy the equations

$$p^2\Phi = 0, \quad \mathcal{D}_a\Phi = 0, \quad \mathcal{D}^\alpha\Phi = 0, \quad \mathcal{D}^{AB}\Phi = 0, \tag{10a,b,c,d}$$

where now

[13] Except for $\mathcal{D}_a$: $\{\chi_\alpha{}^\beta, \mathcal{D}_a\} = -\tilde{v}^\beta_a\mathcal{D}_B v^B_\alpha$. The latter, however, has no effect on the Dirac bracket relations (6).

$$p_\mu = -i\partial/\partial x^\mu, \quad p_\theta^{1,2\alpha} = -i\partial/\partial\theta_\alpha^{1,2}, \quad p_{v,A}^\alpha = -i\partial/\partial v_\alpha^A, \quad p_{\tilde v}{}_\alpha^A = -i\partial/\partial\tilde v_A^\alpha.$$

Eq. (10c) gives

$$\Phi(x, \theta^1, \theta^2, v, \tilde v) = \exp(\theta_\alpha^2 \tilde{p}^{\alpha\beta}\theta_\beta^1)\hat\Phi(x, \theta^1, v, \tilde v). \tag{11}$$

Next, eq. (10b) with an account of (10a), (11), reduces to the form

$$\tilde v_a^\alpha p_{\alpha\beta}(\partial/\partial\theta_\beta^1)\hat\Phi(x, \theta^1, v, \tilde v) = 0. \tag{12}$$

To solve (12) it is convenient to change variables $\theta_\alpha^1 \to \varphi_a, \psi_{\dot a}$ as follows:

$$\varphi^a = [(R^{-1})^{ba}\tilde v_b^\alpha - (S^{-1})^{ba}\tilde v_b^\alpha]\theta_\alpha^1, \quad \psi_{\dot a} = \tilde v_{\dot a}^\alpha\theta_\alpha^1 \quad (R_{ab} \equiv \tilde v_a^\alpha p_{\alpha\beta}\tilde v_b^\beta, \quad S_{ab} \equiv \tilde v_a^\alpha p_{\alpha\beta}\tilde v_b^\beta). \tag{13}$$

With (13), eq. (12) reduces to

$$S_{a\dot b}(\partial/\partial\psi_{\dot b})\hat\Phi(x, \varphi, \psi, v, \tilde v) = 0, \quad \text{i.e. } \hat\Phi = \hat\Phi(x, \varphi, v, \tilde v). \tag{14}$$

In terms of the new coordinates (13), the expressions for $\mathscr{D}^{AB}$ (4) look more complicated:

$$i\mathscr{D}_a{}^b = v_\alpha^b \partial/\partial v_\alpha^a - \tilde v_a^\alpha \partial/\partial\tilde v_b^\alpha - \varphi^b \partial/\partial\varphi^a, \quad i\mathscr{D}_{\dot a}^b = v_\alpha^b \partial/\partial v_\alpha^{\dot a} - \tilde v_{\dot a}^\alpha \partial/\partial\tilde v_b^\alpha + \psi_{\dot a} \partial/\partial\psi_{\dot b},$$

$$i\mathscr{D}_a{}^{\dot b} = v_\alpha^{\dot b} \partial/\partial v_\alpha^a - \tilde v_a^\alpha \partial/\partial\tilde v_{\dot b}^\alpha - (R_{ac}\varphi^c)(S^{-1})^{\dot bd}\partial/\partial\varphi^d + [R_{ac}\varphi^c + (RS^{-1})_a{}^{\dot c}\psi_{\dot c}]\partial/\partial\psi_{\dot b},$$

$$i\mathscr{D}_{\dot a}{}^b = v_\alpha^b \partial/\partial v_\alpha^{\dot a} - \tilde v_{\dot a}^\alpha \partial/\partial\tilde v_b^\alpha - (R^{-1})^{bd}S_{\dot ac}\varphi^c \partial/\partial\varphi^d - (R^{-1}S)^d{}_{\dot a}\varphi^b \partial/\partial\varphi^d + p^2(S^{-1})^{bc}\psi_{\dot c}(R^{-1}\tilde S^{-1})^d{}_{\dot a}\partial/\partial\varphi^d. \tag{15}$$

As a consequence of (15), on-shell φ^a transforms with respect to $\mathscr{D}_A{}^B$ only through itself:

$$\exp\{i\omega^A{}_B\mathscr{D}_A{}^B\}|_{p^2=0}\varphi^a = (\exp\{-\Omega(\omega, v, \tilde v; p)\})^a{}_b\varphi^b, \tag{16}$$

with

$$\Omega(\omega, v, \tilde v; p)^a{}_b \equiv \omega^a{}_b + (S^{-1})^{ac}\omega^d{}_{\dot c}R_{db} + (R^{-1})^{ad}\omega^{\dot c}{}_d S_{\dot cb} + (R^{-1}S)^a{}_{\dot c}\omega^{\dot c}{}_b.$$

Therefore, eq. (10d) yields

$$\hat\Phi(x, \varphi, v, \tilde v) = \hat\Phi(x, \exp\{-\Omega\}\varphi, G^{-1}v, \tilde vG), \tag{17}$$

with $G(\omega)$ as in (8) and $\Omega(\omega, v, \tilde v; p)$ as in (16). Then we can impose on-shell the following covariant reality condition:

$$\hat\Phi^*(x, \varphi, v, \tilde v) = \det^{-1/2}(T(v, \tilde v; p)) \int d^8\varphi' \exp[i\varphi^a T_{ab}(v, \tilde v; p)\varphi'^b]\hat\Phi(x, \varphi', v, \tilde v), \tag{18}$$

where under group transformations generated by $\mathscr{D}_A{}^B$:

$$T_{ab}(G^{-1}(\omega)v, \tilde vG(\omega); p) = T_{cd}(v, \tilde v; p)\Omega^c{}_a(\omega, v, \tilde v; p)\Omega^d{}_b(\omega, v, \tilde v; p). \tag{19}$$

Eq. (19) guarantees that (18) is consistent with (17).

If we now impose the noncovariant light-cone type gauge (9) and choose

$$T_a{}^b(-\tfrac{1}{2}\sigma^\pm, \tilde\sigma^\pm; p) = (p^+)^{-1}\delta_a{}^b \quad (p^+ = (1/\sqrt{2})(p^0 + p^9)),$$

we arrive precisely to the light-cone on-shell description of the linearized $D = 10$ IIb supergravity in terms of an unconstrained superfield $\Phi_{GS}(x, \hat\varphi)$ depending on a single SO(8) (s)-spinor $\hat\varphi^a$ [9]:

$$\Phi_{GS}(x, \hat\varphi) \equiv \hat\Phi(x, \exp\{-\Omega_0\}\varphi, -\tfrac{1}{2}\sigma^\pm, \tilde\sigma^\pm), \quad p^2\Phi_{GS}(x, \hat\varphi) = 0,$$

$$\Phi_{GS}^*(x, \hat\varphi) = (p^+)^4 \int d^8\hat\varphi' \exp[i\hat\varphi^a\hat\varphi'_a(p^+)^{-1}]\Phi_{GS}(x, \hat\varphi'),$$

where $\Omega_0 \equiv \Omega$ $(\omega_0, v, \tilde{v}; p)$ and $G(\omega_0)$ is just the transformation (8) for which the gauge (9) is reached:

$$G^{-1}(\omega_0)v = -\tfrac{1}{2}\sigma^{\pm}, \quad \tilde{v}G(\omega_0) = \tilde{\sigma}^{\pm}.$$

Therefore, we conclude that eqs. (10a), (17)–(19) constitute a Lorentz-covariant on-shell description of $D = 10$ linearized IIb supergravity.

4. To perform second quantization of the superparticle (1), one needs first to construct the first-quantized BRST charge Q_{BRST} and then proceed e.g. along the lines of ref. [10]. It turns out that the $N = 1, 2$ superparticle is a constraint system of second rank (according to the terminology of ref. [7]), i.e. its BRST charge contains a multighost interaction term due to the nontrivial dependence of the structure "constants" of the constraint algebra (6) on the canonical variables. After straightforward (but lengthy) calculations we find

$$Q_{\mathrm{BRST}} = cp^2 + \eta^a \mathscr{D}_a + \mathscr{D}^{\alpha}\chi_{\alpha} + c^A{}_B \mathscr{D}_A{}^B + \mathrm{i}(\partial/\partial\lambda)\partial/\partial\tilde{c} + \mathrm{i}(\partial/\partial\Lambda^A{}_B)\partial/\partial\tilde{c}_A{}^B$$

$$- (\partial/\partial\rho^a)\partial/\partial\tilde{\eta}_a - (\partial/\partial\psi_{\alpha})\partial/\partial\tilde{\chi}^{\alpha} + \mathrm{i}c^B{}_D c^A{}_B \partial/\partial c^A{}_D - 2(\chi_{\alpha}\tilde{v}^{\alpha}_a\eta^a)\partial/\partial c - \mathrm{i}\eta^b c^a{}_b \partial/\partial\eta^a$$

$$+ \eta^b c^a{}_b[\mathrm{i}(\tilde{S}^{-1}\tilde{R})_{\dot{a}}{}^{\dot{d}}\partial\partial\eta^{\dot{d}} + (\tilde{S}^{-1})_{\dot{a}d}v^d_{\alpha}\mathrm{i}(\partial/\partial\theta^1_{\alpha} - \tilde{p}^{\alpha\beta}\theta^2_{\beta})\partial/\partial c] + \tfrac{1}{4}\eta^b c^e{}_j c^{\dot{a}}{}_b(\tilde{S}^{-1})_{\dot{a}e}(S^{-1})^{jd}(\partial/\partial\eta^d)\partial/\partial c,$$

$$(20)$$

where $(c, \tilde{c}, \lambda)$, $(\eta^a, \tilde{\eta}_a, \rho^a)$, $(\chi_{\alpha}, \tilde{\chi}^{\alpha}, \psi_{\alpha})$, $(c^A{}_B, \tilde{c}_A{}^B, \Lambda^A{}_B)$ are the ghosts, antighosts and Lagrange multipliers, respectively, for the first-class constraints in (2).

For the second quantization it is useful to perform the following unitary transformation on Q_{BRST} [11]:

$$Q'_{\mathrm{BRST}} = UQ_{\mathrm{BRST}}U^{-1}, \quad \ln U = -(\ln\alpha)(c\partial/\partial c + \tilde{c}\partial/\partial\tilde{c} - 1), \quad \alpha \equiv (2\lambda)^{1/2},$$

$$Q'_{\mathrm{BRST}} = \alpha^{-1}c[p^2 + \mathrm{i}(\partial/\partial c)\partial/\partial\tilde{c}] + \mathrm{i}(\partial/\partial\alpha)\partial/\partial\tilde{c} + [\text{same as in (20) with } \partial/\partial c \to \alpha\partial/\partial c].$$

Then the BFV hamiltonian [7]

$$H_{\mathrm{BFV}} = \{Q'_{\mathrm{BRST}}, \partial/\partial c\} = \alpha^{-1}[p^2 + \mathrm{i}(\partial/\partial c)\partial/\partial\tilde{c}]$$

has precisely the same form as for the usual bosonic point particle and, accordingly, the second quantized BRST action reads [10]:

$$S = \int d\tau(dz)\Phi^*(\tau, (z))[\alpha\mathrm{i}\partial_{\tau} - p^2 - \mathrm{i}(\partial/\partial c)\partial/\partial\tilde{c}]\Phi(\tau, (z)), \qquad (21)$$

where

$$(z) \equiv (x^{\mu}, \theta^{1,2}, v, \tilde{v}, \alpha, c, \tilde{c}; \eta^a, \tilde{\eta}_a, \rho^a; \chi_{\alpha}, \tilde{\chi}^{\alpha}, \psi_{\alpha}; c^A{}_B, \tilde{c}_A{}^B, \Lambda^A{}_B).$$

For eq. (21) to represent the Lorentz-covariant action for the $D = 10$ IIb supergravity in terms of unconstrained $N = 2$ superfields one still needs to extend off-shell the reality condition (18). At present, unfortunately, we have not found a closed form for the latter.

In conclusion, let us once again stress that, as discussed in section 1 above, Lorentz-covariant treatment of superparticle actions, containing the generator of the local fermionic symmetry as a constraint, is not consistent due to the reducibility of the set of first-class constraints unless one introduces additional (pure gauge) degrees of freedom – the bosonic Lorentz-spinor harmonics, which enable us to select an irreducible subset of covariant constraints. As a result, the rank of the system becomes two and, correspondingly, the BRST charge contains a higher order ghost-interaction term.

A further more important task is to generalize the present formulation for the covariant canonical quantization of the Green–Schwarz superstring [4]. There, similarly, the problem of reducible constraints does arise. Although a Lorentz-covariant separation of the second- and first-class constraints has been performed [12],

the latter appear as projected MW spinors and thus apparently form a reducible set. Also, in the Siegel modification ([1]) of the Green–Schwarz superstring the reducibility of the string generalizations of the constraints $p^2 = 0$ and $\not{p}d = 0$ should be taken into account for a consistent BFV–BRST quantization.

Appendix. The $D = 10$ γ-matrices and $D = 10$ charge conjugation matrix are taken in the following representation:

$$\gamma^\mu = \begin{pmatrix} 0 & (\sigma^\mu)_\alpha{}^\beta \\ (\tilde{\sigma}^\mu)_{\dot\alpha}{}^\beta & 0 \end{pmatrix}, \quad C_{10} = \begin{pmatrix} 0 & C_8{}^{\alpha\beta} \\ (-C_8)^{\alpha\beta} & 0 \end{pmatrix}, \quad C_8^{\mathrm{T}} = C_8^{-1} = C_8,$$

$$\gamma^\pm = (1/\sqrt{2})(\gamma^0 + \gamma^9), \quad (\sigma^\mu)_\alpha{}^\beta (\tilde{\sigma}^\nu)_\beta{}^\gamma + (\sigma^\nu)_\alpha{}^\beta (\tilde{\sigma}^\mu)_\beta{}^\gamma = 2\eta^{\mu\nu}\delta_\alpha{}^\gamma, \quad {}^{(}\not{p}{}^{)} = p^\mu {}^{(}\tilde{\sigma}_\mu^{)}.$$

Indices of $D = 10$ left-(right-)handed MW spinors Φ_α, $\Psi_{\dot\alpha}$ are raised by means of C_8:

$$\Phi^\alpha = (-C_8)^{\alpha\beta}\Phi_\beta, \quad \Psi^\alpha = C_8{}^{\alpha\beta}\Psi_\beta.$$

Analogously,

$$(\tilde{\sigma}^\mu)^{\alpha\beta} = C_8{}^{\alpha\dot\alpha}(\tilde{\sigma}^\mu)_{\dot\alpha}{}^\beta, \quad (\sigma^\mu)_{\alpha\beta} = (-C_8)_{\alpha\beta}(\sigma^\mu)_\alpha{}^\beta.$$

The $D = 8$ charge conjugation matrix may in turn be taken in the form

$$C_8 = \begin{pmatrix} C_6{}^{ab} & 0 \\ 0 & (-C_6)^{\dot a \dot b} \end{pmatrix}, \quad C_6^{\mathrm{T}} = C_6^{-1} = C_6,$$

and the indices of SO(8) (s)-, (c)-MW spinors φ_a, $\psi_{\dot a}$ are raised as

$$\varphi^a = C_6{}^{ab}\varphi_b, \quad \psi^{\dot a} = (-C_6)^{\dot a \dot b}\psi_{\dot b}.$$

References

[1] W. Siegel, Class. Quant. Grav. 2 (1985) L95; Nucl. Phys. B 263 (1985) 93.
[2] H. Terao and S. Uehara, Z. Phys. C 30 (1986) 647;
   P. Townsend, Space-time supersymmetric particles and strings in background fields, DAMPT preprint (1986).
[3] E.S. Sokatchev, Phys. Lett. B 169 (1986) 209; Imperial College preprint ICTP/85-86/21.
[4] M.B. Green and J.H. Schwarz, Nucl. Phys. B 243 (1984) 285.
[5] I.A. Batalin and E.S. Fradkin, Phys. Lett. B 122 (1983) 157;
   I.A. Batalin and G.A. Vilkovisky, Phys. Rev. D 28 (1983) 2567.
[6] C. Becchi, A. Rouet and R. Stora, Ann. Phys. 98 (1976) 287.
[7] E.S. Fradkin and G.A. Vilkovisky, Phys. Lett. B 55 (1975) 224; CERN preprint TH-2332 (1977);
   I.A. Batalin and G.A. Vilkovisky, Phys. Lett. B 69 (1977) 309;
   E.S. Fradkin and T.E. Fradkina, Phys. Lett. B 72 (1978) 343;
   M. Henneaux, Phys. Rep. 126 (1985) 1.
[8] A. Galperin, E. Ivanov, S. Kalitzin, V. Ogievetsky and E. Sokatchev, Class. Quant. Grav. 1 (1984) 469; 2 (1985) 155.
[9] M.B. Green and J.H. Schwarz, Phys. Lett. B 122 (1983) 143;
   L. Brink, M.B. Green and J.H. Schwarz, Nucl. Phys. B 219 (1983) 437.
[10] A. Neveu and P.C. West, CERN preprints TH-4564 (1986); Phys. Lett. B 182 (1986) 343.
[11] L. Baulieu, W. Siegel and B. Zwiebach, University of Maryland preprint UMDEPP 87-71 (1986).
[12] T. Hori and K. Kamimura, Progr. Theor. Phys. 73 (1985) 476.